Multiple Paths FORWARD

Diversifying Mathematics as a Strategy for College Success

Pamela Burdman, Just Equations // Kathy Booth, WestEd
Chris Thorn, WestEd // Peter Riley Bahr, University of Michigan
Jon McNaughtan, Texas Tech University // Grant Jackson, University of Michigan

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With support from the California Community Colleges Chancellor's Office, The James Irvine Foundation, and College Futures Foundation, three organizations — WestEd, Just Equations, and Center for the Study of Higher and Postsecondary Education at the University of Michigan — have joined forces to publish this report. Its goal is to inform California community colleges in their implementation of new student success policies by shedding light on the use of multiple math pathways across the country.

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Just Equations reconceptualizes the role of mathematics in ensuring equal opportunities for students. An independent resource on the role of math in education equity, Just Equations works across educational segments and advances evidence-based strategies to ensure that math policies give all students the quantitative foundation they need to succeed in college and beyond. Just Equations is a project of the Opportunity Institute, in partnership with LearningWorks, Policy Analysis for California Education, the Education Trust—West, and the Campaign for College Opportunity.

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Executive Summary

California community colleges are implementing a new law, AB 705[1], that is intended to improve students' completion of college degrees and credentials. A central objective of the law is to increase students' likelihood of completing math requirements in one year or less. Providing students with multiple pathways to fulfill those requirements is one strategy California community colleges can use to achieve that objective.

Historically, the algebra-to-calculus pathway has been many colleges' default math requirement for students. However, over the last decade it has become clear that this pathway doesn't reflect changes in the types of quantitative skills that students need in their lives and careers. Numerous mathematics-related associations and faculty organizations have advocated diversified math pathways that address the range of undergraduate majors and career goals. These educators have recommended that colleges develop additional non-algebra pathways to ensure that students gain a solid foundation in such important concepts as statistics, modeling, and quantitative reasoning.

The need for alternative math pathways is underscored by research into the factors that limit college completion rates. Both nationally and in California, the vast majority of incoming community college students have been placed into remedial, or developmental, math sequences that emphasize basic algebra and that are designed to prepare students for algebra-intensive pathways. This practice has particularly affected underrepresented-minority students, who are more likely to be placed into remedial sequences. Many students never make it out of the developmental sequence and, thus, are unable to fulfill requirements for graduation or transfer.

In response, postsecondary institutions have begun rethinking their approach to math preparation. In addition to making changes in placement policies and prerequisites, many are offering more than one pathway through mathematics and are working to align those pathways with students' academic and career goals. In one respect, higher education institutions in California have been leaders in this trend: Both community colleges and public universities in the state accept alternative courses such as statistics to fulfill students' math requirements.

However, until recently, the state's four-year institutions specified that community college math courses needed to have a remedial prerequisite — intermediate algebra — in order to qualify as a transfer course. In most colleges, this expectation has been applied to any student placed into remedial math, even if the student intended to take statistics, which doesn't generally assume knowledge of intermediate algebra. As a result, community colleges' work to diversify mathematics pathways in California has focused on general education courses, not remedial sequences. In an effort to preserve students' opportunity to pursue a bachelor's degree, few of the state's community colleges have offered

1 https://leginfo.legislature.ca.gov/faces/billTextClient.xhtml?bill_id=201720180AB705

remedial mathematics pathways designed to prepare students for statistics and quantitative reasoning, despite evidence that such alternative pathways can improve student success. A new, broader definition of quantitative reasoning general education courses that was recently adopted by the California State University (CSU) system eliminates the explicit intermediate algebra requirement, thus freeing community colleges in the state to diversify remedial math courses.

California is one of at least 24 states that are implementing diversified math pathways. These pathways help ensure that students enroll in mathematics courses that are aligned with their program of study; are able to complete a non-remedial course within one year; and benefit from evidence-based curriculum and pedagogy. Preliminary research has found that initiatives to diversify math pathways have yielded two, three, and four times the gateway course completion rates of traditional pathways, often in less time.

The three most common types of mathematics pathways found in other states are a statistics pathway, a quantitative reasoning pathway, and an algebra-based pathway for science, technology, engineering, and math (STEM) majors that require calculus. Other pathways include math for elementary educators, math modeling, technical math, business math, symbolic logic, personal finance, and computer science. In addition, some colleges offer courses associated with specific trades, such as math for clinical calculations.

To identify the most prevalent math pathways in California community colleges, the authors examined the highest-level math courses taken by 900,000 students between fall 2009 and spring 2016. The analysis uncovered a wide variety of offerings, including 11 categories of math that are alternatives to the traditional algebra-based sequences. Overall, transferable alternatives to algebra-based courses accounted for 25 percent of highest-level math completions, with statistics emerging as the most developed of the alternatives. However, almost 50 percent of students only got as far as remedial math. Those students who stopped at remedial math accounted for two thirds of students who dropped out of college, more than half of students who earned a certificate, and 20 percent of students who transferred without an award. These findings are relevant because remedial courses are often misaligned with students' programs of study, focusing on algebra rather than on the types of quantitative skills needed for non-STEM majors.

Though California's higher education systems have not jointly recommended a set of math pathways through two-year and four-year colleges, this report reveals the extent to which the state's community colleges have been active in diversifying their math offerings. Many of the pathways in use in California have parallels to those that have been recognized by other states' higher education systems, as well as to those in CSU's new, broader definition of quantitative reasoning general education courses. Together, AB 705 and CSU's new policy present California community colleges with an opportunity to further diversify their math pathways and ensure that students have the specific quantitative skills they need for future success when they graduate and/or transfer.

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AN OVERVIEW

Multiple Paths Forward

Diversifying Mathematics as a Strategy for College Success

Policy changes by California State University, along with the new AB 705 law, give California community colleges greater ability to diversify math pathways and ensure that students learn the quantitative skills necessary for success.

In response to recommendations from mathematics associations and faculty organizations, and research that shows algebra-intensive math requirements can be a barrier to completion, California has joined more than 20 states in implementing multiple math pathways that align with students' programs of study and enable them to complete a non-remedial math course within a year.

ACROSS THE US ABOUT **HALF** THE **STATES** ARE IMPLEMENTING **MULTIPLE MATH PATHWAYS**

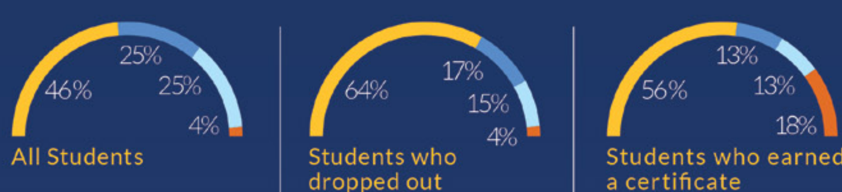
HERE ARE EXAMPLES OF HOW COLLEGES MIGHT ALIGN MAJORS AND PROGRAMS WITH ENTRY-LEVEL MATH COURSES

STATISTICS	Psychology	Social Sciences	Public & Protective Services	Library and Information Services	Media & Communication Health Technologies Journalism Social Work
QUANTITATIVE REASONING	Arts, Humanities & English	Applied Arts and Sciences	Hospitality & Culinary Arts	Agriculture & Natural Resources	
ALGEBRA-TO-CALCULUS	Biology	Engineering & Architecture	Math	Physical Sciences	

CALIFORNIA IS A LEADER IN OFFERING MULTIPLE **PATHWAYS,**

with statistics a common alternative to algebra. But making intermediate algebra a pre-requisite for transferable math has limited the types of quantitative skills students learn. Further diversifying math pathways and aligning pre-requisites can help students leave college with the skills necessary for ongoing study, careers, and life.

Highest-level math course taken by California community college students



A quarter of students take transferable non-algebra courses like statistics

Two thirds of students who drop out stop at remedial math.

Half of certificate earners stop at remedial math.

- REMEDIAL MATH
- COLLEGE ALGEBRA OR ABOVE
- NON-ALGEBRA, TRANSFERABLE TO 4-YEAR INSTITUTIONS
- NON-ALGEBRA, NON-TRANSFERABLE

Percentages based on analysis of courses taken by 900,000 students over 7-year period

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Preface

Fifty years ago, most college students weren't expected to take a mathematics course unless their major required it, and only 15 percent of math chairs at public institutions of higher education favored including a math course as a graduation requirement. By the 1970s, in step with rapid growth in college enrollments, a new emphasis on "general education," and a concern about bolstering the nation's technological prowess, that thinking began to change. A math class was becoming a standard expectation for students seeking two- or four-year college degrees, regardless of their field of study (Burdman, 2015a). By 2010, the vast majority (87 percent) of the nation's universities had a math or quantitative reasoning requirement for graduation (Schield, 2010).

Yet, at the same time, it was becoming clear that such requirements weren't keeping pace with changes in the kinds of quantitative skills that students would need in their lives and careers. In particular, the required courses did not reflect the mounting use, in recent decades, of data and statistics by citizens and consumers, and across a variety of fields. Often, the graduation requirements consisted of either an advanced algebra course or a course for which algebra 2 was a prerequisite. As a result, students' preparation in algebra — typically measured by standardized placement tests — became a common indicator of readiness for college generally. And to ensure readiness, a structure of algebra-intensive remedial courses was developed for those students whose math test scores were deemed too low for them to be placed into a math course required for graduation.

The traditional algebra-to-calculus math sequence was relevant to success in specific fields such as engineering and physics. But it was largely disconnected from most majors, which don't require preparation for calculus. Perhaps as a result, the remedial sequences and the tests responsible for placing students into them weren't enhancing learning for a large percentage of students. In fact, research has made clear that, rather than serving as the foundation for success that they were intended to be, the remedial sequences have functioned as a filter that has prevented many students from advancing in college (Scott-Clayton & Rodriguez, 2012).

Those unintended consequences have been particularly troubling in California, with its diverse population. Underrepresented-minority students, often underserved in the K–12 system, have been more likely than other students to be placed into remedial courses (Ganga, Mazzariello, & Edgecombe, 2018). In California's community colleges, nearly 85 percent of African American and Latino students take remedial math courses, compared to 72 percent of White students and 52 percent of Asian American students (Cal-PASS Plus, 2018). Such placements have been shown to reduce students' likelihood of success in college, and there is some evidence that the effect is even stronger for African American students (Bailey, Jeong, & Cho, 2010).

Underrepresented-minority students have also been more likely to be placed into longer remedial sequences. Compared to 15 percent of White and Asian American college students, about 40 percent of African American students and 30 percent of Latino students have found themselves assigned to classes in arithmetic, further decreasing their chances of college completion (Bahr, Perry, Rosin, Woodward, & Williams, 2010). This may be one reason that efforts to improve education equity have yet to succeed.

In response to growing awareness of curricular misalignment and of the barriers posed by traditional remedial math requirements, postsecondary institutions nationally are rethinking their approach to math preparation. In addition to making changes in placement policies and course prerequisites, they are increasingly focused on diversifying math pathways — offering more than one pathway through mathematics — and ensuring that those pathways align with students' academic and career goals.

Though several leading math pathways models had already been developed in California, in 2017, the state adopted AB 705,[2] a broad overhaul of basic skills education. The new law requires colleges to maximize students' chances of enrolling in and completing a math course appropriate to their education goals within one year of first attempting a math course. Together with parallel changes at the state's public universities, implementation of the law will impact both individual students and the state as a whole, presenting an opportunity to minimize the unintended consequence of math requirements functioning as a barrier to students' college success.

Deepening the use of diversified math pathways is one strategy that California's 114 community colleges can consider adopting to meet their obligations under the new law. Diversifying math pathways is also an integral part of colleges' work to develop "guided pathways," which are described on page 8.

This report, published by WestEd and Just Equations, in collaboration with the University of Michigan's Center for the Study of Higher and Postsecondary Education, highlights the implementation of multiple math pathways. It begins with a discussion of the rationale and context for the multiple math pathways approach, its effectiveness to date, and the range of math pathways that have been conceived and developed around the country. That discussion is followed by an examination of recent mathematics course-taking patterns in California's community colleges, with a goal of informing implementation of AB 705 and highlighting future opportunities for pathways development.

2 https://leginfo.legislature.ca.gov/faces/billTextClient.xhtml?bill_id=201720180AB705

1. Responding to the Challenge of Low Math Success

In addition to addressing the math preparation needs of different disciplines, diversifying math pathways has increasingly been seen by educators as a strategy for improving math achievement. Over the past 10 years, community colleges around the country — along with a growing number of universities — have homed in on mathematics as one of the major obstacles to college completion. Most college and university systems require students to complete a mathematics course before earning a degree. Students who aren't deemed ready for these general education, or gateway, math courses have traditionally been assigned to pre-collegiate mathematics sequences that have been labeled as basic, developmental, or remedial. In recent years, researchers have found that, at best, students assigned to these sequences are no more likely to complete college than similarly prepared students who do not take the sequences. There is also evidence suggesting that the students in the sequences are worse off (Community College Research Center, 2014).

One explanation for this low success is that developmental mathematics courses, especially sequences of two or more courses, delay progress toward a degree and provide more exit ramps for students (Xu & Dadgar, 2017). Other explanations include ineffective courses that discourage students (Grubb, 2011) and placement policies that direct students to remedial courses they may not need (Scott-Clayton, Crosta, & Belfield, 2014). In response to such evidence, college systems have begun compressing, redesigning, or eliminating remedial sequences altogether. In addition, they have worked to improve instruction, strengthen supports for students, and revamp their placement policies to ensure that students don't languish in developmental courses.

These explanations and strategies apply similarly to English and mathematics. Another possible reason for low success in developmental courses applies specifically to mathematics: Traditional developmental mathematics pathways, which emphasize the algebra-intensive content designed to prepare students for calculus, are not relevant to many students' education goals or career paths (Chen & Soldner, 2013). In fact, the algebra content can come at the expense of mathematical topics that would better prepare students for success in college and beyond (Burdman, 2015a).

At stake for students in California and nationally is how they meet the "gateway" math requirement established by most colleges and universities. A student interested in psychology, for example, might benefit more from a statistics course and its relevant prerequisites than from a standard remedial sequence that includes intermediate algebra. Likewise, a quantitative reasoning course that teaches proportional reasoning and dimensional analysis might be most helpful for students interested in fields like allied health, environmental science, and agriculture.

A Backdrop of Innovation and Reform

In response to the poor record of developmental courses and the concern about the relevance of undergraduate math curriculum, a movement has developed to diversify mathematics pathways, with the twin goals of (1) providing students the quantitative skills needed for their chosen field of study and for their roles as citizens and consumers, and (2) ensuring that math requirements don't arbitrarily block students' progress toward a degree.

With respect to general education or gateway math courses accepted for transfer, many higher education systems that once required a college algebra course for every graduate now allow students to take alternative quantitative reasoning courses. California institutions have been ahead of that curve: For at least two decades, the state's public universities have accepted some non-algebra courses, such as statistics, as transferable gateway courses. Likewise, most California community colleges have traditionally offered more than one gateway math course for transfer. (See section 5 for a more detailed examination of course-taking patterns in the California Community Colleges.)

Other policies and practices that have emerged across the country over the past decade reflect the move toward diversification and point to opportunities for California's higher education institutions to further explore. They include:

- Rather than automatically requiring an intermediate algebra course as a prerequisite for all gateway mathematics courses, many systems are tailoring their remedial prerequisites to the gateway math courses that align with students' programs of study.
- The Carnegie Foundation for the Advancement of Teaching developed Statway and Quantway, two-semester sequences that mix developmental and college-level content and that are now being implemented by more than 90 higher education institutions around the country through the Carnegie Math Pathways program[3] (K. Klipple, personal communication, April 26, 2018).
- The California Acceleration Project has developed a model remedial pre-statistics course, as an alternative to remedial algebra, which has been emulated by at least 40 California colleges to accelerate students through college-level statistics courses (Snell, 2018).
- The University of Texas's Dana Center Mathematics Pathways program has supported more than a dozen states in implementing diversified math pathways across two- and four-year institutions, and its model is being used by the majority of institutions in Texas (Getz, 2018).
- The authors of the Common Core State Standards in Mathematics acknowledged that, at the K-12 level, there is some need for differentiating math content by designating some high school algebra standards as being needed only for students going into STEM fields (Burdman, 2015a).

Historically, both the University of California (UC) and California State University (CSU) systems required that general education math courses have an intermediate algebra prerequisite. Thus, for a community college gateway math course to meet the systems' requirements for general education credit, the

3 Carnegie Math Pathways joined WestEd in early 2018.

Definitions

Alternative course or pathway: A course or pathway that differs from the traditional algebra-to-calculus pathway and is designed to serve students in non-STEM fields. An alternative remedial course is a course other than intermediate algebra that prepares students for courses such as statistics or quantitative reasoning.

Gateway math course: A course that meets the requirements for a student's program of study. For a student seeking to transfer to a four-year university, the gateway math course aligns with the general education math requirements of the state's universities. For California community college students earning an associate degree and not transferring, the gateway math course is intermediate algebra or another course at that level.

Math pathway: A gateway math course aligned with a student's program of study or a sequence of math courses whose end point is a gateway math course aligned with a student's program of study.

Pre-collegiate content: Content that traditionally was described as "remedial" or "developmental" and was considered a prerequisite to a general education course. Because, under AB 705, many remedial courses will be replaced by co-requisite courses and other forms of just-in-time support, this report uses the term "pre-collegiate content" to refer to content that in the past has been considered remedial.

Quantitative reasoning: A term often employed to describe the use of mathematical concepts to understand quantitative relationships and solve problems in real-world contexts, it is also an umbrella term used in higher education to describe mathematics and related fields, such as statistics, that foster mathematical literacy. For example, universities often have quantitative reasoning requirements for graduation.

course needed to have intermediate algebra as a prerequisite. The intermediate algebra standard applied to any student who had placed into remedial math, regardless of whether the student's gateway math course or eventual major required the use of algebra. For students earning an associate degree but not transferring, the requirement has been intermediate algebra or another course at the same level.

Concerns that intermediate algebra is not an appropriate course for all students date back at least to 2006, when the community colleges' statewide academic senate promised the colleges' Board of Governors that it would actively promote alternative courses to intermediate algebra (Walton, 2013). In fact, some of California's community colleges have been at the forefront of developing alternative developmental pathways, particularly as preparation for introductory statistics classes. But until recently, efforts to expand these pathways were stymied by university prerequisite policies, because college leaders were wary of adopting programs that could imperil students' transfer options (Burdman, 2015b). Most colleges have yet to diversify remedial mathematics pathways at sufficient scale to reach the majority of students in non-STEM programs. Also, most offer only a limited number of sections, with the bulk of enrollments remaining in traditional algebra-based pathways. A 2017 Public Policy Institute of California study found that only 2 percent of developmental enrollments were in statistics pathways (Rodriguez, Johnson, Mejia, & Brooks, 2017).

New California Policy Context

Recent policy changes are creating fertile ground for further diversifying math pathways. A new law, AB 705 (Seymour-Campbell Student Success Act of 2012, 2017), signed by Governor Jerry Brown in October 2017, mandates that colleges use multiple measures, including students' high school records, to determine whether students need remedial coursework. Under AB 705, community colleges are expected to maximize the chances that students who seek to transfer can enter and complete a transfer-level math course within a one-year time frame. (The same is true for English courses.) For students who are pursuing a degree or certificate but are not planning to transfer, the goal is for them to enter, and complete within one year, a college-level math course relevant to their field of study. In fact, under the new law, colleges seeking to place a student into a remedial course need to demonstrate that the student is "highly unlikely" to succeed in a gateway course without it.

With AB 705, California community colleges will no longer be able to offer lengthy developmental sequences or to require math courses that are not aligned with students' majors. Though not mandated by the law, diversifying mathematics pathways will be a key strategy for colleges as they work to comply with the new law and to ensure that mathematics requirements don't arbitrarily prevent students from entering an academic program.

Coinciding with passage of AB 705 have been CSU policy changes making it clear that courses other than intermediate algebra are now acceptable as preparation for transfer-level mathematics courses that are not algebra-intensive, such as statistics. In 2016, CSU's academic senate convened the Quantitative Reasoning Task Force, which supported the concept of developing new mathematics pathways and provided some guidance for doing so. A year later, CSU Chancellor Tim White issued two executive orders. One[4] (White, 2017a) eliminated stand-alone remedial coursework and placement tests. The other[5] (White, 2017b) stipulated that:

- students may take courses other than traditional mathematics to fulfill their general education quantitative reasoning requirement (a requirement that applies to both freshmen and transfer students), and
- prerequisites for such courses should reflect only the actual skills and knowledge required to be successful in the course.

Like AB 705, the new CSU policies seek to enhance equity by eliminating remediation as a barrier to students' completing their education. The policies take effect in fall 2018, following faculty professional development during 2017/18 to develop new pathways. Now that CSU and community colleges have adopted similar frameworks for accelerating students into college-level courses, transfer policy is no longer an obstacle for community colleges interested in implementing alternative math pathways. Unlike CSU's prior intermediate algebra prerequisite, the UC system had a broader requirement of intermediate algebra "or its equivalent." UC's recently revised transferable course

4 https://www.calstate.edu/eo/EO-1110.html

5 https://www.calstate.edu/eo/EO-1100-rev-8-23-17.html

articulation guidelines[6] specify that pre-collegiate requirements for statistics should be consistent with the content of the Common Core math standards (Regents of the University of California, 2015). In effect, proficiency in intermediate algebra is no longer an explicit requirement for every community college student seeking to transfer, though it remains a prerequisite for STEM-oriented programs.[7]

Math Pathways Linked to Several Aspects of AB 705 Reforms

While this report focuses on math pathways, it's important to note that implementation of diversified math pathways intersects with other reforms mandated or encouraged by AB 705. These reforms align with the national trend toward reducing community college remedial math course-taking (Blair, Kirkman, & Maxwell, 2018) and may help colleges meet the requirement to ensure that students can complete a gateway math course within a year of first attempting one.

Multiple measures placement. Under AB 705, colleges are required to base placement decisions on more than one factor, including students' high school records. This approach has been adopted by numerous states, based on research showing that, compared to placement tests, high school grades better predict students' performance in college (Burdman, 2015c). In mathematics, students' academic goals can be another factor in determining appropriate course placement. Placing students into math courses that appropriately prepare them for their field of study requires that colleges offer more than one pathway through mathematics. In addition, colleges are barred from requiring prerequisites unless students are "highly unlikely" to succeed in a subsequent course without it.

Co-requisite courses. Co-requisite courses are gateway courses that incorporate pre-collegiate content (or are paired with pre-collegiate course sections) that students may need to be successful in the course. To accelerate students' progress into gateway math courses, students may be placed into co-requisite courses or other forms of gateway courses with just-in-time support. Rather than requiring that a student complete a prerequisite *before* entering the gateway course, a co-requisite model entails students completing pre-collegiate content *within* or *concurrent with* the gateway course. Ensuring that all pre-collegiate content is aligned with the student's general education (or gateway) math course is another strategy for accelerating the student's progress and meeting AB 705's requirement that the support provided increase students' likelihood of passing the course.

Stretch courses. Two-semester pathways that integrate pre-collegiate and gateway math content, sometimes called "stretch courses," are a common model for statistics and quantitative reasoning courses. They are effectively two-semester co-requisites. Whether these approaches are considered to be AB 705–compliant will depend on research about students' likelihood of success in gateway math. Some colleges are working to shorten two-semester pathways to single-course co-requisites.

6 https://www.ucop.edu/transfer-articulation/transferable-course-agreements/tca-policy/regulations-by-subject-area.html#s

7 The CSU Chancellor's Office recently conducted a review of a subset of associate degrees for transfer that had accepted a statistics course to fulfill the math requirement and determined that no change was needed despite the removal of intermediate algebra as a prerequisite (A. Wrynn, personal communication, April 11, 2018).

The move to align students' mathematics preparation and skills with their majors and careers also dovetails with a national movement to promote "guided pathways." The guided pathway model[8, 9] (Community College Research Center, n.d.) is designed to help students reach their transfer and career goals by:

- providing structured opportunities to select a program of study within a general discipline;
- ensuring that students have clarity regarding course sequences and key milestones within their chosen program,
- redesigning math and English requirements to align with student pathways,
- providing stronger supports for students, and
- ensuring opportunities for active learning.

All of California's 114 community colleges are implementing a guided pathways approach through the California Community College Guided Pathways[10] project (California Community Colleges Chancellor's Office, 2017). Colleges have received multi-year grants to help them adopt and scale up effective guided pathways practices. Diversifying mathematics sequences and ensuring that students have math options aligned with their program of study and transfer goals support successful guided pathways by eliminating obstacles for students.

The following sections take a closer look at the context for and development of diversified mathematics pathways around the country.

- Section 2 examines the context in which mathematics pathways have multiplied.
- Section 3 defines mathematics pathways and shares outcomes to date of several pathways initiatives.
- Section 4 more closely analyzes the range of pathways that are in various stages of implementation in higher education systems across the country.
- Section 5 examines the existing mathematics landscape in the California community colleges, with the intent of shedding light both on the current status of mathematics pathways and on opportunities for diversification.

8 WestEd is a partner in the California Guided Pathways project, which is supporting 20 colleges in implementing the national guided pathways model.

9 https://www.aacc.nche.edu/wp-content/uploads/2018/01/12PathwaysModelDescriptionFinal1616.pdf

10 http://cccgp.cccco.edu/

2. Evolving Perspectives from the Math Community

Dramatic changes in the role of mathematics departments, along with a growing realization that math was serving as a gatekeeper to students' achievement of their education aspirations, have led to significant soul-searching across the discipline. The introduction of math requirements for most college students in the latter part of the 20th century gave rise to new demands on math departments. Enrollment in courses such as statistics and computer science began to outpace enrollment in traditional sequences. At the same time, remedial enrollments mushroomed, apparently because the introduction of general education math requirements brought more students into math departments, which, in turn, led to widespread use of standardized tests to assess their readiness for traditional math courses (Burdman, 2015a; Fey, Albers, Fleming, & Lindquist, 1981). Over the past decade, research began showing that remedial math courses were not contributing to student success at community colleges and, in fact, were exacerbating equity gaps in college completion (Mejia, Rodriguez, & Johnson, 2016; Community College Research Center, 2014; Scott-Clayton & Rodriguez, 2012).

The resulting sense of urgency, combined with growing awareness of the varied contexts in which mathematics and quantitative reasoning are used across fields, put pressure on the mathematics field to modernize its undergraduate offerings. In response, leaders within the mathematics community have increasingly stressed the need to ensure that math education remains relevant to students and responsive to real-world applications of mathematics. In recent years, various national associations have formed a chorus of voices advocating for reform, with an emphasis on broadening the range of math pathways available to students.

In 2011, a joint committee of the National Council of Teachers of Mathematics (NCTM) and the Mathematical Association of America (MAA) convened a panel that questioned the notion that, as described by NCTM's president at the time, "high school students should take or be prepared to take calculus, and that the path to calculus needs to be paved with frequent and repetitive overdoses of algebra" (Shaughnessy, 2011, para. 3). The president further wrote, "This is an out-of-date, wasteful, and repetitive transition path for our students. Worse, it does nothing to improve our students' disposition toward mathematics. When students are confined to this tunnel of repetitive algebra, they never have opportunities to experience the beauty, excitement, power, or usefulness of mathematics" (Shaughnessy, 2011, para. 4).

The panel proposed four pathways that high schools and colleges should consider offering: One would focus on data analysis, combinatorics, and probability and emphasizing how to quantify uncertainty and analyze numerical trends; a second would focus on statistical thinking and understanding the need for data and the importance of data production; a third would be grounded in linear algebra, and, in it, students would work with multivariable problems and learn general-purpose matrix methods; and the fourth would focus on multivariate applications of calculus and statistics (Shaughnessy, 2011).

In 2012, the Obama Administration's Council of Advisors on Science and Technology raised a similar concern about undergraduate mathematics courses:

> *Introductory mathematics courses often leave students with the impression that all STEM fields are dull and unimaginative, which has particularly harmful effects for students who later become K–12 teachers. Reducing or eliminating the mathematics-preparation gap is one of the most urgent challenges — and promising opportunities — in preparing the workforce of the 21st century.* (President's Council of Advisors on Science and Technology, 2012, p. iv)

The high cost of developmental education and the private sector's needs were both cited as evidence of the imperative to change undergraduate math education.

Leading mathematicians and math associations arrived at the notion that an element of the desired transformation was a more expansive set of undergraduate mathematics offerings — offerings that better reflect how mathematics is used in a variety of disciplines and careers. In its 2013 report *The Mathematical Sciences in 2025,* the National Research Council noted:

> *The educational offerings of typical departments in the mathematical sciences have not kept pace with the large and rapid changes in how the mathematical sciences are used in science, engineering, medicine, finance, social science, and society at large. This diversification entails a need for new courses, new majors, new programs, and new educational . . . partnerships . . . Different pathways are needed for students who may go on to work in bioinformatics, ecology, medicine, computing, and so on. It is not enough to rearrange existing courses to create alternative curricula.* (p. 10)

As diversification gained currency, the traditional reliance on intermediate algebra as a prerequisite for gateway mathematics courses also came under scrutiny.

In 2014, the American Mathematical Association of Two-Year Colleges (AMATYC) adopted a position statement noting that "prerequisite courses other than intermediate algebra can adequately prepare students for courses of study that do not lead to calculus" (para. 6).

Around the same time, leading mathematicians came together to found a new organization, Transforming Post-secondary Education in Mathematics (TPSE Math). The group aimed to galvanize mathematics faculty around a set of innovations to ensure that mathematics instruction fosters "the mathematical knowledge and skills necessary for productive engagement in society and in the workplace" (TPSE Math, 2015, p. 2).

One of TPSE Math's three strategic priorities is to address "insufficiently differentiated math pathways" (p. 2). The association noted that this condition "stems from the fact that students are not always prioritized; math departments are relatively rigid in the area of instruction and serve as service departments rather than true partners to other disciplines; and implementation of cross-departmental efforts requires a different 'collective action and responsibility' approach rather than just innovations limited to individual classrooms" (pp. 2–3).

In 2015, the MAA and the four other leading mathematical sciences associations collectively reviewed the curricular recommendations they had issued in recent years. The resulting report, *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025* (Saxe & Braddy, 2015), was part of an initiative to chart a way forward for undergraduate mathematics. It is noteworthy that the authors found that all of the guides "call for multiple pathways into and through mathematical sciences curricula" (p. 13). Some of these pathways, the report noted, "should include early exposure to statistics, modeling, and computation" (p. 13). In addition, they asserted,

> *There is a call to provide mathematically substantive options for students who are not headed to calculus. These entry courses should focus on problem solving, modeling, statistics, and applications. Current college algebra courses serve two distinct student populations: (1) the overwhelming majority for whom it is a terminal course in mathematics, and (2) the relatively small minority for whom it is a gateway to further mathematics. Neither group is well-served by the traditional version of the college algebra course. There is a mismatch between a curriculum designed to prepare students for calculus and the reality that only a small proportion of these students subsequently enroll in calculus. We acknowledge the need to focus on the calculus sequence and ensure that pathways to it remain a high priority, as calculus is central to most further study in the mathematical sciences, but it behooves us to develop curricula effective for the majority of the population as well.* (p. 13)

As strong as they are in making the case for diversifying undergraduate mathematics pathways, the associations' position statements are short on specifics. In general, they support instructional practices similar to those advanced by the Common Core State Standards in mathematics. The statements generally focus on broad content areas, not on specifying learning outcomes or offering concrete guidance to colleges or systems looking to do so. Much of the work of defining and designing pathways has been left to specific pathways initiatives, state math task forces, and higher education systems.

3. Defining and Assessing Math Pathways

Calls for innovation in college math curriculum have intersected with efforts by state systems and foundations to improve college outcomes, especially for community college students. Together, they have spurred numerous national and state initiatives to engage in the work of defining, building, and evaluating diversified math pathways. At a national level, both Carnegie Math Pathways and the University of Texas at Austin's Charles A. Dana Center have model pathways that have been adopted by institutions and states around the country. Within California, the California Acceleration Project has also developed design principles that faculty from more than 40 of the state's 114 community colleges have used to redesign their math sequences.

The seeds for these projects were planted about 10 years ago, as research was beginning to show how mathematics requirements could thwart students' progress toward a degree. Awareness of those findings caused those who were leading initiatives for improving community college student success to take a hard look at remedial math sequences.

The research was revealing. Among other things, it showed that it was not uncommon for community college students to be assigned to three or four remedial math courses, starting as low as arithmetic, and that there were even colleges with five levels of remediation. The length of the sequences was shown to be a barrier to completion, because the time involved provides opportunities for students to "stop out"[11] even if they pass courses within a sequence (Bailey, Jeong, & Cho, 2010; Xu & Dadgar, 2017).

These findings prompted efforts at the K–12 level to improve students' readiness for college, as well as work by colleges to improve the effectiveness of remedial coursework in both mathematics and English. While scrutinizing the length of the math sequences, reform leaders also began to discuss the sequences' content. The emphasis on algebra 2 as a common measure of proficiency for college-level mathematics, regardless of students' majors, seemed to be a mismatch for the programs pursued by most students. Against this backdrop, the early pathways initiatives took root, with significant support from private foundations as well as state systems.

Nationally, in addition to pathways provider organizations, policy and advocacy groups such as Complete College America and, in California, the Campaign for College Opportunity have promoted adoption of diversified mathematics pathways, as does the guided pathways movement.

11 Stopping out refers to the phenomenon of college students putting their education on hold with the intention of returning and completing a degree, but also alludes to the fact that many students do not ultimately return.

More than a dozen state postsecondary systems have convened math task forces or similar efforts, many of them supported by Texas's Dana Center, to develop pathways that best serve their students.

Definitions of math pathways can be found in task force recommendations, as well as in such documents as system-level policies, cross-segmental mathematics discipline agreements, and state articulation agreements. In many cases, the definitions were developed by committees made up of, or including, math faculty. For example, in 2013 the Ohio Board of Regents established the Ohio Mathematics Initiative, comprising mathematics faculty from two- and four-year institutions, and charged it to "develop expectations and processes that result in each campus offering pathways in mathematics that yield: (1) increased success for students in the study of mathematics; (2) a higher percentage of students completing degree programs; and (3) effective transferability of credits for students moving from one institution to another" (Ohio Mathematics Initiative, 2014, p. 2).

As noted earlier, a mathematics pathway can be defined as a gateway math course or set of math courses whose end point is a gateway math course aligned with a student's intended program of study.

The Dana Center promotes four principles of math pathways[12] (Dana Center Mathematics Pathways, n.d.):

1. All students, regardless of college readiness, enter directly into mathematics pathways aligned to their programs of study.
2. Students complete their first college-level mathematics requirement in their first year of college.[13]
3. Strategies to support students as learners are integrated into courses and are aligned across the institution.
4. Instruction incorporates evidence-based curriculum and pedagogy.

Alignment is a central feature of math pathways, with the intent of ensuring that students are learning math content that fits with their chosen discipline or major. Students in psychology, for example, benefit from studying statistics. Some biology departments, such as University of California Los Angeles's, have developed courses that mix calculus and statistics content to replace traditional calculus. Community college students pursuing certificates in construction crafts need grounding in geometry. Ideally, the appropriate math pathway for a given major would be determined via a content analysis.

While a pathway can consist of a single gateway mathematics course, a fully elaborated pathway encompasses the preparation some students may require for success in that course. Pre-collegiate content, which can be differentiated to align with a gateway math course, comes in various forms:

- a stand-alone remedial course
- a set of developmental, or remedial, modules
- supplementary instruction

12 https://dcmathpathways.org/dcmp/dcmp-model

13 Note that in California, that there is a distinction between transfer-level math and a math course accepted for an associate degree.

- a co-requisite course (or set of paired courses) in which the pre-collegiate content is taught along with the college-level content
- a two-semester “stretch” course that combines pre-college and college-level content over two semesters

Ideally, pathways implementation is paired with placement policies that align with the pathways. Because traditional tests that are used to determine whether students need pre-collegiate coursework have emphasized algebra competency (CalPASS Plus, 2014; Burdman, 2015c), moves to de-emphasize placement test scores are compatible with development of alternative math pathways.

While pathway development efforts continue to be evaluated, research to date has shown the potential of diversified math pathways to yield significant improvements in student success. Research has found that pathways initiatives have yielded two, three, and four times the success rates of the traditional pathway, with students completing a college-level course in less time. (See the box, Adoption of Alternative Mathematics Pathways Associated with Dramatic Improvements in Student Success, below.)

The research to date raises several questions. Because colleges historically assigned many students to college algebra, for example, the extent to which success of pathway models should be attributed to the alternative pre-collegiate content or, instead, to changes in general education requirements is not fully clear. A contributing factor may be that fewer students are assigned to college algebra courses, which often have lower pass rates than courses that emphasize statistics and quantitative reasoning.

Adoption of Alternative Mathematics Pathways Associated with Dramatic Improvements in Student Success

A multivariate logistic regression analysis of the California Acceleration Project (CAP), controlling for prior ability and other factors, found that community college students in a statistics pathway were 4.5 times as likely to complete a transfer-level course as similar students placed into traditional remediation. The improvement held across all placement levels and demographic groups (Hayward & Willett, 2014).

In a study by the Carnegie Foundation, researchers found large positive effects for the two-course Statway sequence. More than half of community college Statway students completed their gateway math requirement in their first year, compared with fewer than 20 percent of non-Statway students. As a result, the Statway students accumulated more college-level credits in the following year (Huang, Norman, & Yamada, 2018).

In the other Carnegie pathway, more than half of Quantway students enrolled in gateway math by the following year, compared with just 30 percent of students in traditional remediation. The Quantway students also tended to be more successful in the gateway course, earning an average 2.22 GPA, compared to a 2.06 for non-Quantway students (Huang, Norman, & Yamada, 2018).

The final report on the Dana Center Mathematics Pathways, showing outcomes such as completion of gateway mathematics, will not be complete until 2019, but preliminary findings show that students in these pathways were more likely to succeed in their developmental math coursework than students in traditional remedial courses (Dana Center Mathematics Pathways, n.d.).

Single-Semester Pathways. While most of the research in this area has focused on two-semester pathways, a couple of studies have looked at single-semester co-requisite versions and found positive outcomes.

A randomized controlled study at the City University of New York (CUNY) assigned students to one of three groups: (1) traditional remedial elementary algebra; (2) traditional remedial elementary algebra with a co-requisite workshop led by undergraduate students; and (3) college-level statistics with a co-requisite workshop led by undergraduates. The students assigned to the third group passed their course at a rate of 56 percent, compared with 39 percent and 45 percent for the first and second groups, respectively (Logue, Watanabe-Rose, & Douglas, 2016).

Also, five Statway colleges that experimented with a single-semester high-contact-hour co-requisite reported positive outcomes, according to a Carnegie Foundation report. While half of standard Statway students achieved college math credit within one year, 67 percent of accelerated Statway students reached that goal (Huang, Norman, & Yamada, 2018).

Likewise, Tennessee's higher education system, in its work to pioneer the co-requisite model, incorporated diversified pathways at the college level. Students there experienced a four-fold improvement in outcomes during the first year of full implementation (Tennessee Board of Regents, n.d.a).

Impact Across Races and Ethnicities. Though more research is needed on how alternative remedial math pathways impact students across races and ethnicities, studies suggest that they have potential to reduce achievement gaps. The 2014 CAP study pointed to the possibility that CAP's design principles could be a key component of a strategy to reduce the achievement gap for African Americans, and it recommended further study (Hayward & Willett, 2014). The 2018 Carnegie study of Statway noted that improved outcomes emerged across genders and races/ethnicities (Huang, Norman, & Yamada, 2018). Tennessee's approach did not completely eliminate the achievement gap, but it did narrow it considerably: Minority students saw a six-fold improvement in outcomes, compared to a four-fold improvement for other students (Tennessee Board of Regents, n.d.a). However, the Public Policy Institute of California (PPIC) observed the opposite in California: While accelerated math pathways improved outcomes for all demographic groups, the improvement for African American students was less dramatic than for White students (Rodriguez, Johnson, Mejia, & Brooks, 2017).

Longer-Term Outcomes. While early studies have only addressed completion of a gateway math course, evidence is beginning to emerge about the effects of mathematics pathways on longer-term outcomes. According to a 2017 Carnegie study, compared with traditionally remediated students, Quantway students were nearly 45 percent more likely to earn an associate degree or transfer to a four-year university within four years, and Statway students were about 35 percent more likely to transfer, but they earned associate degrees at rates similar to those of other students (Norman, 2017).

In their analysis of alternative remedial math pathways at 45 of California's community colleges, PPIC researchers found that 27 percent of students in these pathways earned a credential or transferred to a four-year university within three years, compared to 13 percent of students in a traditional remedial pathway (Rodriguez, Johnson, Mejia, & Brooks, 2017).

And when it comes to success in subsequent math-related courses, a small unpublished study at San Jose State University found that Statway students who continued on to other quantitative courses outperformed non-Statway students in terms of pass rates and grades in those courses (Branz & Heil, 2016).

Lastly, in an unpublished paper, the CUNY authors reported that students in the co-requisite statistics course completed gateway math courses and accumulated credits at higher rates than other students. In addition, students in the statistics course were 50 percent more likely than students in the other groups to graduate within three years (Watanabe-Rose, Logue, & Douglas, 2017).

Also, it should be emphasized that many of these efforts integrate math pathways with other innovations, such as pedagogical improvements, accelerated or compressed curriculum, co-requisite courses, and placement reforms, often through changes to system and state policy. For example, Tennessee's four-fold increases in student success in gateway mathematics occurred after institutions in that state adjusted placement standards, eliminated college algebra as a requirement for all students, and implemented co-requisite approaches to remediation (Burdman, 2017). In addition, the efforts tend to employ evidence-based approaches to professional development, such as networked improvement communities. The Carnegie Math Pathways, Dana Center, and CAP approaches also integrate recommendations from learning science to address students' social-emotional needs. These strategies have particular potential to advance student learning, including for those from underrepresented groups, who tend to experience traditional math classrooms as unsupportive and alienating (Blackwell, Trzesniewski, & Dweck, 2007; Haynes, Perry, Stupnisky, & Daniels, 2009).

The strong outcomes cannot be attributed to curricular diversification alone. The initiatives generally facilitate students' starting in higher-level math, which shortens the pipeline and contributes to stronger outcomes. Given the powerful effects across a wide range of initiatives, it is clear that diversifying mathematics pathways doesn't just make sense conceptually, it is also a component of strategies to improve student mastery of quantitative skills and boost on-time completion rates.

4. Multiple Pathways and Learning Outcomes

Higher education systems in at least 24 states have implemented or taken steps toward implementing multiple math pathways, and additional efforts are underway at individual colleges. In developing their pathways, many of these states have been supported by one or more national organizations. The Dana Center has worked with 15 states to support scaled implementation of mathematics pathways. Carnegie Math Pathways is working with 90 colleges across 18 states to implement its Statway and Quantway pathways. Complete College America has also supported several states in adopting math reforms, including math pathways.

In many places, the pathways strategy began with community colleges, but, increasingly, state-level math task forces are developing pathways that apply to both two-year and four-year institutions. Those states that began in the two-year sector, such as Colorado and Washington, have expanded to work with four-year institutions in order to ensure that courses can smoothly articulate across campuses for students seeking to transfer.

Common Pathways Options

The most common pathways across states mirror the three that were initially piloted by the two national pathways initiatives, Carnegie Math Pathways and the Dana Center:

- an **algebra-based pathway** that leads to calculus and is commonly used as preparation for STEM courses,
- a **statistics pathway**, and
- a **quantitative reasoning pathway.**

But there are variations and overlaps in states' choices of pathways (see the box, Range of Pathways, on p. 18). An algebra-based pathway is a constant, but the new "STEM prep" courses tend to be more-modern alternatives to college algebra. Across states, the most common non-algebra-based pathway appears to be quantitative reasoning, which also goes by such names as Liberal Arts Mathematics and Math in the Modern World. Quantitative reasoning pathways typically include some statistics content, in addition to covering such content as numerical reasoning, proportional reasoning, and algebraic reasoning, often using real-world problems. Quantitative reasoning courses also tend to incorporate instruction in mathematical modeling (for example, linear, piecewise, and exponential models), but one state — Georgia — has a distinct modeling pathway in addition to its quantitative reasoning pathway.

(For more information on these pathways, see the appendix, Learning Outcomes for Common Math Pathways.)

Some states, including Indiana, Georgia, and Nevada, don't have a statistics pathway. At the other end of the spectrum, the statistics pathway in California community colleges is highly developed and has long been accepted by public universities as a general education math course. (See section 5, "Examining Math Course-Taking in California Community Colleges," on p. 23.)

While a range of pathways exist, the Dana Center has recommended that states choose somewhere between three and seven distinct pathways. The aim is to provide enough options while also ensuring transparency and facilitating transfer and articulation across campuses and systems (A. Getz, personal communication, April 2, 2018).

Range of Pathways

At least 10 states have recognized or recommended a set of math pathways for two-year and/or four-year institutions. In addition to the most common pathways — the traditional algebra-based STEM pathway, the statistics pathway, and the quantitative reasoning pathway — a scan of state and national initiatives and of institutional quantitative reasoning policies has identified at least eight other math pathways. Some were developed based on different disciplines' demand for more customized quantitative preparation.* The first seven pathways listed below (in the left-hand column) have clearly elaborated learning outcomes, at least in some states. Typically, these learning outcomes are based on guidance from national disciplinary societies as well as on the use of the Dana Center and/or Carnegie models. A few other recognized pathways do not yet have elaborated learning outcomes. (See the appendix, Learning Outcomes for Common Math Pathways, for a list of learning outcomes associated with specific math pathways where available).

Most Common Math Pathways:

- STEM (algebra-based) Pathway
- Statistics Pathway
- Quantitative Reasoning Pathway

Other Pathways With State-Level Outcomes:

- Math for Elementary Educators
- Math Modeling
- Technical Math (generally non-transferable)
- Business Math

State- or System-Recognized Pathways Without State-Level Outcomes:

- Symbolic Logic
- Personal Finance
- Computer Science

Examples of Other Growing Pathways (No State-level Outcomes)

- Data Science
- Career-based Pathways

* For guidance on how to develop learning outcomes for math pathways, see Dana Center Mathematics Pathways, "A Process for Success: Developing and Supporting Student Learning Outcomes for Multiple Math Pathways" (https://dcmathpathways.org/sites/default/files/resources/2017-03/A%20Process%20for%20Success%20-%20Developing%20and%20Supporting%20Student%20Learning%20Outcomes%20for%20Multiple%20Math%20Pathways.pdf) (Dana Center Mathematics Pathways, 2017).

Other Pathway Possibilities

States and colleges have developed other mathematics pathways that are aligned with programs of study. For example, some colleges offer courses associated with specific trades.

The Indiana community college system has developed a non-transferable course that teaches real-life math applications based on technical concepts (Indiana Commission for Higher Education, 2015). Community College of Denver, among its eight entry-level pathway courses, offers Math for Clinical Calculations[14] (Community College of Denver, n.d.). Designed for students in the health disciplines, it includes problem-solving skills related to drug dosage and intravenous fluid administration. As these career-based math courses generally are not designed to be transferable, the colleges don't need to worry about articulation. However, most of the state math task forces have targeted transferable math courses as the end points of their recommended pathways.

In California, recent decisions by CSU may open a door for more-diverse transferable math pathways. For example, the system's executive order indicating that CSU accepts non-traditional mathematics content to meet its quantitative reasoning requirement mentioned two additional courses for meeting that requirement: personal finance and computer science. To date, neither one has been specifically recognized by any of the state math task forces.

According to a catalog review by CSU's Chancellor's Office[15] for the General Education Mathematics and Quantitative Reasoning committee (California State University, 2018), only CSU East Bay was offering a personal finance pathway. The review covered courses offered before fall 2017, when CSU dropped its intermediate algebra prerequisite. The CSU East Bay course was described in the catalog[16] as "Principles and practices of money management, consumer credit, savings, investments, taxation, and consumer protection." Looking out of state, Community College of Denver's Financial Mathematics[17] course covers topics such as "pricing, taxes, insurance, interest, annuities, amortization, investments using financial calculators and spreadsheets" (para. 1). At the University of Missouri, a personal and family finance[18] course meets the math reasoning proficiency requirement. Its topics are "individual and family finance, with particular emphasis on financial planning, savings, insurance, investments, taxes, use of credit, and financial aspects of housing" (para. 3).

Computer science as a math pathway is the subject of some debate. Some institutions explicitly exclude computer science courses from meeting their quantitative reasoning requirements, not viewing these courses as sufficiently mathematical. A number of highly selective institutions — such as Harvard, Columbia, and UC Berkeley — do accept computer science courses for transfer. Prior to fall 2017, six CSU quantitative reasoning courses involved some computing, according to CSU's catalog review (see table 1, on p. 20).

14 http://catalog.ccd.edu/programs-courses/courses/mat/

15 http://calstate.edu/app/mathqr/ (Click on third arrow [CSU GE Math/Quantitative Reasoning Courses], then download "More Courses.")

16 http://catalog.csueastbay.edu/preview_course_nopop.php?catoid=4&coid=12955 (Use search function on site to find catalog.)

17 http://catalog.ccd.edu/search/?search=MAT+112

18 http://catalog.missouri.edu/courseofferings/finpln/

TABLE 1.

Courses That Involved Computing and That Met CSU's Quantitative Reasoning Requirement (Prior to Fall 2017)

CSU Campus	Course	Department
Channel Islands	Scientific Computing	Mathematics
Fresno	Statistical and Computer Applications in Criminal Justice	Criminal Justice
Pomona	Logic and Computing	Philosophy
San Diego	Computational Thinking	Computer Science
San Marcos	Computer Science I Media-Propelled Computational Thinking	Computer Science

Source: California State University (2018) (http://calstate.edu/app/mathqr/). (Click on third arrow [CSU GE Math/Quantitative Reasoning Courses], then download "More Courses.")

The existence of these courses, as well as the executive order's explicit recognition of computer science, raises the possibility that such courses will multiply at CSU campuses and community colleges that serve as transfer institutions.

Lastly, a burgeoning trend at four-year universities is to offer data science courses to meet general education requirements in math or quantitative reasoning. A few years ago, after more than half of its freshman class signed up for courses in computer science and statistics, UC Berkeley piloted a data science course[19] that blends computer science and statistics. It became the fastest growing new course in campus history. The course teaches much of the content of an introductory statistics course, along with Python programming skills and "powerful understanding of key ideas in computing" (University of California at Berkeley, n.d., para. 8).

Advising and Pathway Access

The state-level adoption of pathways can facilitate policy solutions to common dilemmas. There are questions about how students enter a pathway and how to determine which one best aligns with their program of study. Ineffective advising has been an issue. Georgia, for example, had several gateway math courses approved, but the state's math task force discovered that many students were being advised to take college algebra even when their majors did not call for it (University System of Georgia Mathematics Task Force, 2013).

The same concern arose in Colorado, where roughly two thirds of community college students were enrolled in college algebra, compared with less than half at four-year institutions. In 2015, the Colorado Math Pathways Task Force noted that,

> *It is likely that students are being advised into college algebra in community colleges because advisors see that as a 'safe' course to enroll in if a student does not know where they might transfer in the*

19 https://data.berkeley.edu/education/foundations

> *future or what their program of study will be . . . Even in majors in which any college math course is allowable for the degree program, such as English or History, we think that a well-designed course in the QuantThinkingPath [that is, quantitative reasoning] would be a much more relevant educational experience and be better connected to their program of study than college algebra.* (p. 11)

Colleges and states have addressed this issue in a few ways. Ivy Tech has set quantitative reasoning as the default math pathway for students. In its 2013 recommendations, a group of math faculty from two- and four-year institutions, making up the University System of Georgia Mathematics Task Force, likewise recommended *against* treating college algebra as a default course:

> *The practice of using College Algebra as a proxy for general quantitative ability or to ensure that students can later transfer to a STEM major must end. System data suggest that placement in College Algebra . . . is a major contributor to student failure. Furthermore, the broad audience in College Algebra makes it challenging to organize this important course as a true stepping stone to Calculus.* (p. 6)

Georgia is one of several states that specify alignment[a] between math pathways and college majors (University System of Georgia Mathematics Task Force, 2013). The Colorado Math Pathways Task Force used meta majors — broad areas of study that encompass related majors — as the basis for its recommendations and its updated Statewide Degree Transfer Agreements in 2015:

1. CalcPath — STEM, certain health sciences, and many business programs
2. StatPath — Social & Behavioral Sciences
3. QuantThinkingPath — Arts & Humanities

Colorado's policy also allows for exceptions for such majors as early childhood education, elementary education, architecture, and business. (Not surprisingly, two of these — elementary education and business — have been chosen by some states as distinct pathways.)

In other cases, states and systems simply discourage colleges from requiring college algebra unless it's aligned with the students' program, or they require or incentivize colleges to adopt more than one pathway. Massachusetts' math pathways subcommittee recommended[b] four math pathways for adoption by all colleges (Massachusetts Department of Higher Education, 2018). Adoption of the five pathways recognized by the Washington Math Pathways to Completion Task Force[c] (2017) is voluntary; however, the state has asked colleges to commit to implementing at least three. The Missouri Department of Higher Education (2015) requires colleges to have at least two math pathways, and it has a set of approved pathway courses, but doesn't specify which courses colleges are required to offer. Ohio incentivizes adoption of approved math pathways by including these pathways in the state's module of courses approved for transfer (Ohio Mathematics Initiative, 2016).

[a] http://www.completegeorgia.org/math-recommendations

[b] https://dcmathpathways.org/sites/default/files/resources/2018-01/Designing%20Math%20Pathways%20Report_FINAL_0.pdf

[c] https://dcmathpathways.org/sites/default/files/resources/2017-03/Task%20Force%20Report_Washington%20Math%20Pathways%20to%20Completion%20Task%20Force.pdf

Math Pathways and Remedial Requirements

Rethinking remedial requirements is also a key component of effective pathways reform. Many math pathways states extend the pathways approach to pre-collegiate material. In these cases, prerequisites or co-requisites are tailored to the specific pathway. Some states have explicitly stated that intermediate algebra or algebra 2 should not be treated as a prerequisite for a quantitative reasoning course. Ohio's task force advises against using algebra 2 in this way, but doesn't explicitly forbid it (Ohio Mathematics Initiative, 2014).

Aligning preparation with the college-level pathway course is intended to streamline students' progress through a gateway math course by eliminating content that isn't relevant for a student's program of study, while preparing the student to be successful in the college-level course.

Colorado's community college system specifies a quantitative literacy preparation course for its non-algebra pathway. Some states don't diversify their pre-college expectations by pathway (Colorado Math Pathways Task Force, 2015). Tennessee has a common set of pre-collegiate outcomes for its math co-requisite courses, while noting that students pursuing algebra-intensive courses require additional curriculum that should be provided at the college level (Tennessee Board of Regents, n.d.b). Indiana similarly allows for additional preparation for STEM fields (Indiana Commission for Higher Education, 2015). Georgia has a foundations course for all students, but is in the process of phasing it out, effectively eliminating stand-alone remediation (University System of Georgia Mathematics Task Force, 2013).

5. Examining Math Course-Taking In California Community Colleges

Unlike in some states, California higher education institutions have for years offered several types of gateway courses in addition to standard college algebra and other calculus-track courses, but, until recently, the extent of these "algebra alternative" courses has not been known. A 2016 report[20] by CSU faculty's Quantitative Reasoning Task Force found that among CSU's non-transfer students, about 42 percent take a math course outside the traditional STEM or algebra-based pathway (Academic Senate of the California State University Quantitative Reasoning Task Force, 2016). However, that report was only able to identify community college courses *approved* for transfer, not the extent to which community college students transferring to CSU actually *took* those courses.

To get a fuller picture of math course-taking among the state's community college students, the authors of this report, *Multiple Paths Forward,* analyzed a data set provided by the California Community Colleges Chancellor's Office. The goal was to identify the types of mathematics courses taken by more than 900,000 students over a seven-year period (from fall 2009 to spring 2016), including the highest-level courses they completed, including remedial courses. (See Methodology at end of section.) The study revealed several issues that are important to consider as colleges revise their math pathways in the context of AB 705.

California Is Offering a Broad Array of Quantitative Pathways

The analysis found a wide variety of math courses offered across the California colleges, indicating that there is broad acceptance, by colleges and students, of courses outside traditional algebra-based sequences. The scope of content is similar to that of the math offerings at institutions across the country that are described in section 4, as well as to the offerings at CSU, the state's most common transfer destination. However, patterns of math course enrollment varied by students' college outcomes.

The data showed 25 general categories of math courses, including 4 remedial math categories and 11 categories of alternatives to traditional algebra-based pathways. Among the 11 alternative math categories, the courses in 4 of them are generally transferable in that they meet general education requirements for public universities. Courses in the other 7 categories, which the authors are describing as primarily non-transferable, typically do not meet university general education requirements, though some may be transferable as electives. (See the box, Math and Quantitative Reasoning Courses in California Community Colleges, on p. 24.)

20 https://www.asccc.org/sites/default/files/V.%20G.%20QRTF%20Final%20Report%2008-01-2016.pdf

Among courses in the alternative pathways, statistics emerged as the most widespread. The transferable statistics course was by far the most common highest-level alternative math course.[21] That trend held true for non-transfer students earning associate degrees and/or certificates. Overall, transferable alternatives accounted for 25 percent of highest-level math completions, whereas the non-transferable alternatives constituted only 4 percent.

Math and Quantitative Reasoning Courses in California Community Colleges

The authors' investigation found 25 categories of courses across California's 114 community colleges. Among these, the authors classified 11 as "alternative" courses, in that they are not traditional algebra-based courses. These 11 were further divided into four classification.

Traditional Remedial Math Courses

- arithmetic
- pre-algebra
- elementary algebra
- intermediate algebra*

**Note that intermediate algebra is considered below transfer-level, but meets the requirements for an associate degree.*

Non-Transferable Alternative Courses (i.e., Non-Algebra-Based Courses That Do Not Fulfill General Education Transfer Requirements)*

- plane geometry
- symbolic logic
- technical, industrial, trade, and shop math (e.g., Trade Math, Shop Calculations, Technical Math, Mathematics for Water and Wastewater Treatment, Refrigerator Systems Calculations, Technical Math for Electronics, Welding Shop Math)
- business math (e.g., Business Math, Modern Merchandising Math)
- healthcare math (e.g., Math for Healthcare Professionals, Dosage Calculations)
- math for teaching in early childhood education (e.g., Math Curriculum for Children Teaching Early Childhood Math)
- restaurant math (e.g., Culinary Math)

**These courses generally do not satisfy the general education quantitative reasoning requirement at UC and CSU, though some of them are transferable as electives.*

21 Other states seem to be following this trend, as statistics enrollments in community colleges nearly doubled from 2010 to 2015 (Blair, Kirkman, & Maxwell, 2018).

Transferable Alternative Courses (i.e., Non-Traditional Courses That Fulfill General Education Transfer Requirements*)

- general education math for students who are not math majors (e.g., Liberal Arts Math, Survey of Modern Math)
- general education math for teachers (e.g., Structures and Concepts in Elementary Math, Math for Prospective Teachers)
- finite math
- statistics (including courses taught by math and social science departments)

**These courses generally satisfy the general education quantitative reasoning requirement at UC and CSU, though not necessarily requirements for specific majors — with the exception that math for teachers does not satisfy the UC requirement. Note that Finite Math, often taken by business majors, includes algebra topics, but does not typically lead to calculus.*

Transferable Algebra-Based Courses (i.e., Traditional Courses That Fulfill General Education Transfer Requirements)

- applied calculus
- college algebra
- trigonometry*
- pre-calculus
- discrete math
- linear algebra
- calculus I
- calculus II
- calculus III
- differential equations

**UC does not accept a stand-alone trigonometry class for general education.*

Math Course Enrollments Vary by Student Completion Outcomes

The analysis found notable differences in math course-taking patterns across student completion outcomes, as shown in table 2 and described below:

For transfer students who earn community college certificates and degrees, statistics is the most common alternative to algebra-track courses. Among students who transferred with a credential, those taking non-algebra alternatives outnumbered those taking traditional algebra courses. A full 53 percent of these students transferred with a non-algebra alternative as their highest-level math course, with 85 percent of those courses in statistics (see table 2, on p. 27). This exceeds the proportion of CSU students who took an alternative course to fulfill their general education math requirement (Academic Senate of the California State University Quantitative Reasoning Task Force, 2016).

A significant number of students who transfer without a credential never get past remedial math before transferring. Students transferring without a credential were less likely to have taken an alternative course. Only 33 percent of these students completed a transferable alternative, of which 83 percent were statistics courses. Compared with students earning community college awards, these students were more likely to transfer before taking a class categorized as a gateway math course. Fully 20 percent of these students had a remedial course as their highest-level successful math enrollment, whereas only 3 percent of students transferring with a credential fell into this category. Given that UC and CSU require completion of a general education math course for students transferring, it is likely that many of these students were transferring to institutions other than UC or CSU. Among students who did move beyond remedial math, college algebra or a more advanced algebra-based course was the most common highest-level math completion. This pattern was experienced by 46 percent of students transferring without a credential, similar to the 44 percent share of students transferring with credentials.

Transferable alternatives are a common option for degree-earners. Non-algebra course alternatives that meet university general education requirements were also common among students earning associate degrees without transferring. In fact, these courses constituted the most common category of highest-level math course completion. Among these students, 44 percent took a transferable alternative as their highest-level math course (87% of these were statistics courses), compared with just 26 percent taking a traditional algebra-based course, such as college algebra. In addition, 26 percent of associate degree completers showed a remedial math course as their highest-level math completion. The gateway course required for the associate degree is intermediate algebra (classified as a remedial course) or its equivalent. The pattern was roughly similar among students who earned both associate degrees *and* certificates, except that more of these students enrolled in a non-transferable alternative course as their highest-level enrollment.

Non-transferable alternative math pathways predominantly serve students in certificate programs, but certificate completers are more likely to stop at remedial math. Certificate completers were far more likely to show a non-transferable alternative course as their highest-level course. In fact, 18 percent of students who succeeded in at least one quantitative course and who completed a certificate exclusively (without an associate degree or transferring to a four-year institution) completed one of these courses. These courses were most common in fields such as engineering and industrial technologies, as well as business and management. Another 13 percent had a highest-level quantitative course that fell within the transferable alternatives. However, both of these figures pale in comparison to the 56 percent of certificate completers whose highest-level quantitative course completion was remedial in nature. (As noted in the conclusion, further analysis is needed here.)

The majority of students who drop out never get past remedial math. Among students who leave community college without a certificate or degree and do not transfer to a four-year college, 64 percent only got as far as courses in the developmental sequence (including intermediate algebra).

TABLE 2.

Highest-Level Quantitative Courses Successfully Completed by California Community College Students, 2009–2016, by Student Outcome

Highest-Skill Quantitative Course Successfully Completed	No Credential and No Transfer	Certificate Only	Associate Only	Certificate and Associate	Transfer Only	Credential and Transfer	All Outcomes
remedial math (including intermediate algebra)	64%	56%	26%	27%	20%	3%	46%
non-transferable alternatives to algebra-based pathways	4%	18%	3%	7%	1%	0%	4%
transferable alternatives to algebra-based pathways	15%	13%	44%	39%	33%	53%	25%
college algebra or above	17%	13%	26%	27%	46%	44%	25%
total	100%	100%	100%	100%	100%	100%	100%

Notes: Non-transferable alternatives to algebra-based pathways include plane geometry; symbolic logic; technical, industrial, trade, and shop math; business math; healthcare math; math for teaching in early childhood education; and restaurant math. Transferable alternatives to algebra-based pathways include general education math for students who are not math majors; general education math for teachers; finite math; and statistics.

Percentages may not add to 100 due to rounding.

Remedial Requirements May Be Misaligned with De Facto Math Pathways

Nearly 30 percent of the highest-level math completions in the 900,000-student data sample are in alternative pathways, the vast majority of them in statistics, with such courses taken by more than half of students who transferred with credentials. Yet, as other research has shown, about 75 percent of community college students take remedial courses (CalPASS Plus, 2018), the vast majority of those courses designed to prepare students for algebra-intensive courses, not for statistics. This raises questions about the relevance of the pre-collegiate preparation students typically receive. Given that the majority of students who drop out never take a non-remedial math course, AB 705, as well as guided pathways implementation, presents opportunities to reconsider pre-collegiate math requirements.

This misalignment may explain why statistics appears to be the area with the greatest innovation in pre-collegiate preparation. Though this analysis did not address the prevalence of alternative remedial courses, the Public Policy Institute of California looked at the range of pre-statistics courses being offered in the system. Those authors found that 59 community colleges — more than half of the colleges in the system — offer some form of pre-statistics course, according to 2017/18 course catalogs (Rodriguez, Mejia, & Johnson, 2018). However, as mentioned earlier, pre-statistics courses constituted only about 2 percent of enrollments statewide.

Methodology

This study investigated the variety of mathematics and quantitative reasoning courses in the course-taking pathways of California community college students. The analysis focused on first-time students who entered between fall 2009 and spring 2016, and who reported a valid Social Security number at college entry (N = 1,961,653). Fall 2009 marked the implementation of intermediate algebra or an equivalent course as the math requirement for associate degrees (May, 2017). Because the study focused on the system's semester-based community colleges, the analysis excluded the state's three community colleges that operate on a quarter system.

The analysis looked at the course-taking, credential completion, and transfer outcomes of these students through the end of fall 2017, which means that all students were observed for at least two years. The analysis focused on students who attempted at least one math course (including quantitative courses taught outside math departments). Slightly less than two thirds (63 percent; N = 1,243,532) of the students in the sample attempted at least one math or quantitative reasoning course within the period of observation, and almost three quarters (73 percent; N = 905,183) of those students were successful in at least one of those courses. The study focused on students who succeeded in at least one math or quantitative course, passing it with a grade of C or better, Pass, or Credit. The analysis excludes students who did not attempt such a course, and also excludes students who attempted but did not succeed in such a course.

The objective of the analysis was to investigate the relationship between students' highest-level successfully completed quantitative course and selected credential completion and transfer outcomes. Doing so required identifying and categorizing every quantitative course taken by students in the sample. The study ultimately identified 25 categories of quantitative courses represented in the data. (See the box, Math and Quantitative Reasoning Courses in California Community Colleges, on p. 24.)

Then a hierarchical order was imposed on these categories, from arithmetic (the lowest) through calculus III (the highest), which allowed us to determine the highest-level math course that each student had successfully completed with a grade of C or better, Pass, or Credit. We then determined which of the following mutually exclusive education outcomes was experienced by each student within the observation period:

1. was not awarded a community college credential and did not transfer to a four-year institution,
2. was awarded a community college certificate only,
3. was awarded an associate degree only,
4. was awarded a community college certificate and an associate degree,
5. transferred to a four-year institution but was not awarded a community college credential, or
6. transferred to a four-year institution and was also awarded a community college credential.

The analysis then calculated the distribution of students' highest-level successfully completed math or quantitative course within each of the six outcomes. The distributions were calculated for the whole sample of students who successfully completed at least one math or quantitative course, and for the subsets of men versus women and students of historically advantaged racial/ethnic groups versus students of historically disadvantaged racial/ethnic groups. Finally, we examined the distribution of students' highest-level successfully completed math or quantitative course by program of study for students who were awarded a certificate and, separately, for those who earned associate degrees.

The data set for this study will be used to conduct further studies to allow more nuanced understanding of students' engagement with quantitative reasoning coursework. These studies will include a more granular analysis of the categories of math courses, as well as development of multidimensional measures of course-taking.

6. Conclusion: Suggested Next Steps in Multiple Pathways Research

While California's higher education systems have not jointly developed a strategy for diversifying math pathways across two-year and four-year colleges, the state's community colleges have been active in diversifying their gateway math offerings, as revealed by the findings in this report. Many of those alternative pathways have parallels to those recognized by higher education systems in other states and also correspond to courses accepted under CSU's new, broader definition of quantitative reasoning. The findings in this report demonstrate that large numbers of transfer students and those pursuing associate degrees have been taking transferable alternatives to the algebra pathway, and that, to a lesser extent, non-transferable options have been available to serve students in certificate programs.

But even as California institutions have offered a variety of gateway math courses, past policies have hindered development of alternative pathways at the remedial level. Recently, as universities have broadened their general education requirements, community colleges have begun diversifying remedial math pathways as well. CSU general education policy now allows the transfer of quantitative reasoning courses without an explicit intermediate algebra prerequisite. UC's requirement is intermediate algebra or an equivalent course. Nevertheless, alternative options are far less developed at the remedial level than at the general education level.

As colleges implement AB 705 and seek to maximize students' chances for success in higher education, the findings in this report suggest several areas for further consideration and research:

How do community college students access math pathways, and is that access equitable? As colleges adopt diversified math pathways it is important that students' access be driven by their individual aspirations.[22] Given the history of tracking in education, and its impact on equity, there is a risk of non-algebra pathways being perceived as less prestigious alternatives. Algebra-based pathways, and STEM careers, should be open to students regardless of race or gender. Achieving that goal will likely require intentional strategies, including disaggregating data to monitor enrollment patterns, appropriate advising, and recruiting of students into pathways. Such strategies are especially important given some evidence showing that African American and Latino students are more likely than White and Asian American students to underestimate their proficiency in math (Fong & Melguizo, 2016).

22 This point is affirmed by The Dana Center Mathematics Pathway, "Call to Action", 2016a (https://dcmathpathways.org/sites/default/files/resources/2016-11/The%20Case%20for%20Mathematics%20Pathways.pdf)

Future research and inquiry should examine:

- Enrollment patterns in various math pathways along race and gender lines,
- Differences in availability of math pathways across colleges,
- Student perspectives on diversified math pathways, and
- Policies for informing and advising students about their math pathway options, including alignment of pathways with majors and the use of default requirements.

How do colleges balance the trade-offs between providing a diverse range of math pathways and needing to offer a coherent set of options to students and to articulate those options with other institutions? This report identifies a wider range of math pathway options than all colleges may be able to provide. Colleges' choice of offerings may depend on institutional size and character, but their choice may also need to account for the priorities of transfer institutions in their region.

Areas for future inquiry include:

- General education math offerings at UC and CSU campuses,
- Prospects for expanding pathways that are less prevalent in California than in some other states (e.g., quantitative reasoning), as well as adopting other relatively unexplored math pathways, such as data science, personal finance, and computer science (which are general education courses at some UC and CSU campuses), and
- How alternative math pathways are being integrated into the requirements for associate degrees for transfer.

Colleges value autonomy, but a degree of cross-institutional alignment is important to ensure smooth transitions across colleges and segments. In the past, lack of such articulation has created obstacles to reform efforts in community colleges (Burdman, 2015b). Intersegmental dialogue about these issues at the state and regional levels has begun and will be important to continue.

Does the current range of non-transferable math pathways meet the needs of students in certificate programs? As compared with students who transfer or earn associate degrees, far fewer certificate-earning students take an alternative course as their highest-level math completion. Instead, a large proportion of certificate-completing students take remedial courses. To shed light on which math courses (if any) best support these students in their programs of study, it would be important to better understand the available courses.

Future inquiry could provide:

- A more granular picture of the range of non-transferable options across colleges, as well as enrollment patterns for these courses, and
- Information about the content of these courses and how the courses align with various certificate programs.

There has been very little available research on non-transferable math pathway alternatives. Better understanding California colleges' approach to these courses would be useful within the state and beyond.

Does diversifying math pathways enhance educational outcomes, as intended? The intention of multiple math pathways is to better align students' math requirements with their educational goals to support their success and eliminate arbitrary requirements. While the research described in section 3 suggests that it does so, it will be important to continue to study this question, including with respect to pre-collegiate content.

Future inquiry should examine:

- The extent to which diversification (including the alignment of pre-collegiate requirements with math pathways) contributes to improved longer-term outcomes,
- The extent to which diversification can help reduce equity gaps, and
- Conditions and strategies that enhance these possibilities, including the potential benefit of bridges between pathways for students who switch majors.

How can alternative remedial pathways inform colleges' implementation of AB 705? Under AB 705, many remedial courses may be eliminated in favor of co-requisite courses and other just-in-time approaches. Because most community colleges in California already offer more than one type of gateway math course (Rodriguez, Mejia, & Johnson, 2018), colleges may benefit from examining the appropriate pre-collegiate content for each math pathway. To comply with AB 705, colleges should ensure that pre-collegiate content is aligned with and necessary for gateway math course success.

As mentioned in the previous section, at least half of California's colleges have offered some form of pre-statistics course. Examining the content of these courses may inform efforts to design new models that enable students to pass the gateway courses, such as co-requisite courses. PPIC has also reported that about 29 percent of statistics pathway programs in 2013/14 either had no prerequisite or had an arithmetic prerequisite (Rodriguez, Johnson, Mejia, & Brooks, 2017). Exemplars among these alternative pre-collegiate offerings may emerge for colleges to study as they develop their innovations.

Future inquiry should also examine:

- In addition to statistics pathways, what other alternative or contextualized remedial courses are available, and is there evidence for their effectiveness?

What are the implications of diversified math pathways for students' preparation in high school? Given that math course-taking has traditionally helped shape students' post-high school-options, new approaches to mathematics at California's community colleges can inform decisions about students' preparation in high school. As the 2011 National Council of Teachers of Mathematics/Mathematical Association of America panel envisioned, diversified pathways at the postsecondary level may lead to innovation in K–12 as well (Shaughnessy, 2011). For example, discussions are underway about the number of math courses students take in high school. California requires just two years of math for high school graduation, fewer than are required in 44 other states, though nearly two thirds of California districts require an additional year (Gao, 2017). The Common Core State Standards, adopted eight years ago in California, include three years (through algebra 2), as do the UC and CSU admissions requirements. However, the majority of UC and CSU students take four years of math during high school, and CSU's Academic Senate has proposed[23] making that a requirement (California State University, 2018).

Future inquiry should examine:

- How high school math course-taking and performance relate to students' community college pathway choices and other outcomes, and
- To what extent implementation of diversified math pathways in community colleges lends itself in California and across the country to replication in high school.

Efforts to diversify mathematics pathways have shown notable potential for improving student outcomes, especially in connection with other innovations. Diversification is also integral to other initiatives designed to improve student success, such as guided pathways. California colleges are implementing an array of new strategies — aligned with newly adopted policies — to expand students' options for getting the math they need in order to attain their education goals and their career goals. Further research to assess those efforts, study students' outcomes and experiences, and learn about the inter-segmental implications of math pathways will help guide effective implementation.

23 https://www.calstate.edu/acadsen/Records/Resolutions/2015-2016/documents/3244.shtml

Appendix: Learning Outcomes for Common Math Pathways

STEM/Algebra-Based Pathway

As a reference point in developing learning outcomes for college algebra, many states have used the 2007 College Algebra Guidelines[24] from the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (Ganter & Haver, 2011). However, as the National Council of Teachers of Mathematics/Mathematical Association of America report described in section 2 explains, there is a growing concern in the mathematics community that college algebra as a course does not have a natural constituency (Saxe & Braddy, 2015). Leading math associations note that college algebra is not an effective terminal course for most students in the humanities and social sciences. Thus, its use as a default general education requirement is in question, while pre-calculus appears to be a better option for students in STEM fields or disciplines such as economics that rely on calculus. Virginia's Community Colleges (n.d.) have chosen pre-calculus as the core STEM pathway. The learning outcomes for pre-calculus 1[25] are below:

Upon completing the course, the student will be able to:

Relations and Functions

- Distinguish between relations and functions.
- Evaluate functions both numerically and algebraically.
- Determine the domain and range of functions in general, including root and rational functions.
- Perform arithmetic operations on functions, including the composition of functions and the difference quotient.
- Identify and graph linear, absolute value, quadratic, cubic, and square root functions and their transformations.
- Determine and verify inverses of one-to-one functions.

Polynomial and Rational Functions

- Determine the general and standard forms of quadratic functions.

24 https://www.maa.org/sites/default/files/pdf/CUPM/crafty/CRAFTY-Coll-Alg-Guidelines.pdf
25 https://docs.google.com/document/d/11iFDn1Zb0puIcBnOP5ZXjV5vNkKzg9hWUAtz7cWShik/edit

- Use formula and completing the square methods to determine the standard form of a quadratic function.
- Identify intercepts, vertex, and orientation of the parabola and use these to graph quadratic functions.
- Identify zeros (real-valued roots) and complex roots, and determine end behavior of higher order polynomials and graph the polynomial, and graph.
- Determine if a function demonstrates even or odd symmetry.
- Use the Fundamental Theorem of Algebra, Rational Root test, and Linear Factorization Theorem to factor polynomials and determine the zeros over the complex numbers.
- Identify intercepts, end behavior, and asymptotes of rational functions, and graph.
- Solve polynomial and rational inequalities.
- Interpret the algebraic and graphical meaning of equality of functions ($f(x) = g(x)$) and inequality of functions ($f(x) > g(x)$)

Exponential and Logarithmic Functions

- Identify and graph exponential and logarithmic functions and their transformations.
- Use properties of logarithms to simplify and expand logarithmic expressions.
- Convert between exponential and logarithmic forms and demonstrate an understanding of the relationship between the two forms.
- Solve exponential and logarithmic equations using one-to-one and inverse properties.
- Solve application problems involving exponential and logarithmic functions.

Systems of Equations and Inequalities

- Solve three variable linear systems of equations using the Gaussian elimination method.

To achieve the above objectives, the support course will cover appropriate topics such as those suggested below in both planned review and just-in-time remediation.

Statistics Pathway

The 2016 Guidelines for Assessment and Instruction in Statistics Education College Report[26] (GAISE College Report ASA Revision Committee) published by the American Statistical Association, incorporate nine learning goals. Many of the state and national pathways initiatives have designed their curricula to align with the guidelines:

1. Students should become critical consumers of statistically based results reported in popular media, recognizing whether reported results reasonably follow from the study and analysis conducted.

26 http://www.amstat.org/asa/files/pdfs/GAISE/GaiseCollege_Full.pdf

2. Students should be able to recognize questions for which the investigative process in statistics would be useful and should be able to answer questions using the investigative process.
3. Students should be able to produce graphical displays and numerical summaries and interpret what graphs do and do not reveal.
4. Students should recognize and be able to explain the central role of variability in the field of statistics.
5. Students should recognize and be able to explain the central role of randomness in designing studies and drawing conclusions.
6. Students should gain experience with how statistical models, including multivariable models, are used.
7. Students should demonstrate an understanding of, and ability to use, basic ideas of statistical inference, both hypothesis tests and interval estimation, in a variety of settings.
8. Students should be able to interpret and draw conclusions from standard output from statistical software packages.
9. Students should demonstrate an awareness of ethical issues associated with sound statistical practice.

Quantitative Reasoning Pathway

Ohio developed quantitative reasoning learning outcomes[27] (Ohio Mathematics Initiative, 2015) with a goal of reflecting the recommendations of the Mathematical Association of America's Undergraduate Programs and Courses in the Mathematical Sciences: CUPM (Committee on the Undergraduate Program in Mathematics) Guidelines for College Algebra[28] (Ganter & Haver, 2011, pp. 45-47). The outcomes were developed with the support of staff from the Dana Center and with the participation of faculty teaching Carnegie's Quantway, so the outcomes appear to be representative of those being implemented nationally.

1. Numeracy: Students will develop and use the concepts of numeracy to investigate and explain quantitative relationships and solve problems in a variety of real-world contexts.

    1.1 Solve real-world problems requiring the use and interpretation of ratios in a variety of contexts: Parts to whole comparisons, converting decimals to percentages and vice versa, quantifying risks by calculating and interpreting probabilities, rates of change, and margins of error.

    1.2 Solve real-world problems relating to rates of change, distinguishing between and utilizing models that describe absolute change and relative change including growth and decay.

27 https://www.ohiohighered.org/sites/ohiohighered.org/files/uploads/transfer/documents/OTM/TMM011%20Quantitative%20Reasoning%20FINALIZED%20v2-%2012-21-2015.pdf

28 https://www.maa.org/sites/default/files/pdf/CUPM/crafty/introreport.pdf

1.3 Compare and contrast statements which are proportional and those that are not by applying proportional reasoning appropriately to real-world situations such as scaling, dimensional analysis, and modeling.

1.4 Demonstrate numerical reasoning orally and/or by writing coherent statements and paragraphs.

2. Mathematical Modeling: Students will make decisions by analyzing mathematical models, including situations in which the student must recognize and/or make assumptions.

   2.1 Create and use tables, graphs, and equations to model real-world situations including: using variables to represent quantities or attributes, estimating solutions to real-world problems using equations with variables, identifying pattern behavior, identifying how changing parameters can affect results, and identifying limitations in proposed models.

   2.2 Model financial applications such as credit card debt, installment savings, loans, etc., and calculate income taxes.

   2.3 Create basic linear and exponential models for real-world problems and be able to choose which one is most appropriate for a given context and describe the limitations of the proposed models.

   2.4 Use basic logarithm properties to address questions (regarding time periods, etc.) arising in real-world situations modeled exponentially.

   2.5 Explain and critique models orally and/or by writing coherent statements and paragraphs.

3. Probability and Statistics: Students will use the language and structure of statistics and probability to investigate, represent, make decisions, and draw conclusions from real-world contexts.

   3.1 Critically evaluate statistics being presented in the media, journals, and other publications, including evaluating the research methodology, critiquing how the author(s) came to their conclusions, identifying sources of bias, and identifying confounding variables. Students will be able to critically evaluate sampling strategy, the impact of sample size, correlation versus causation, and any inferences made.

   3.2 Summarize and interpret datasets with regard to shape, center, and spread. Use both graphical and numerical information. Use statistics appropriate to the shape. Students will be able to compare two or more datasets in light of this type of information.

   3.3 Create visual representations of real-world data sets such as charts, tables, and graphs and be able to describe their strengths, limitations, and deceptiveness.

   3.4 Calculate probabilities and conditional probabilities in real-world settings, and employ them to draw conclusions.

   3.5 Justify decisions based on basic statistical (probabilistic) modeling orally and/or by writing coherent statements and paragraphs.

Elementary Education Pathway

There is variation in this area, with some states, such as Massachusetts (Massachusetts Department of Higher Education, 2018, p. 8), requiring college algebra. The Conference Board of the Mathematical Sciences (2012) has issued its own recommendations[29] for preparation of elementary teachers. Through its Math Pathways Initiative, Missouri's Department of Education (n.d.) has developed the set of learning outcomes[30] shown below for preparation of elementary school teachers. Approval of these outcomes is pending as of the date of this report.

I. The Real Number System

Students will be able to explain the real number system and evaluate operations. Specifically, students will be able to:

- Categorize numbers and apply appropriate properties to mathematical statements.
- Find the least common multiple and greatest common factor and illustrate their role in various real number operations.
- Examine and compute various computational algorithms for each of the four basic operations in base ten as well as other bases.
- Utilize the fundamental theorem of arithmetic and discuss its importance in the real number system.
- Explain the four basic operations and use them in computations with integers, fractions, and decimals.
- Use order of operations.
- Solve problems involving percent, ratios, and proportions.

II. Foundations of Algebra

Students will be able to evaluate the foundations of algebra through number, operations, and algebraic thinking. Specifically students will be able to:

- Analyze patterns and functions, including arithmetic and geometric sequences, through the use of words, symbols, tables, graphs, and algebraic notation.
- Model and solve mathematical and real-world problems using linear and quadratic equations and their graphs.
- Solve problems, express identities, and calculate with algebraic notation.

29 https://www.cbmsweb.org/archive/MET2/Chapter_4.pdf

30 https://dhe.mo.gov/documents/MMPTElementaryEducationPathwayoutlineformat.pdf

III. Informed Decision-Making

Students will generate informed decisions through the use of probability and statistics. Specifically, students will be able to:

- Organize and analyze data using descriptive and inferential statistics.
- Formulate and solve problems using experimental the theoretical probabilities (as appropriate).
- Apply counting techniques and principles to find probabilities, including compound events.
- Demonstrate knowledge of sampling to draw inferences.
- Construct and interpret graphical displays of data.

IV. Basic Euclidean Geometry

Students will examine Euclidean geometry. Specifically, students will be able to:

- Solve mathematical and real-world problems.
- Perform calculations involving various two-dimensional and three-dimensional objects.
- Use coordinate geometry.
- Create geometric constructions using a compass and a straightedge to justify geometric definitions, theorems, and postulates.
- Apply concepts of motion in two-dimensional space through transformations.

Mathematical Modeling Pathway

The Society for Industrial and Applied Mathematics has issued a set of guidelines[31] for mathematical modeling instruction and assessment, including a chapter on undergraduate education. Georgia's higher education institutions have adopted math modeling more extensively than most other systems. When the state's mathematics task force went to work, an Introduction to Mathematical Modeling course had already been approved. However, the course was underutilized, because the majority of students were being placed into college algebra (University System of Georgia Mathematics Task Force, 2013). Now the introductory course is one of two non-STEM pathway options for students in Georgia. The learning outcomes below are from a system-approved online version[32] of the course (University System of Georgia, n.d.).

Introduction to Functions

1. Understand the concept of a function.
2. Understand the concept of domain and range of a function.

31 http://www.siam.org/reports/gaimme-full_color_for_online_viewing.pdf

32 https://ecore.usg.edu/img/pdf/syllabi/MATH1101-Syllabus.pdf

3. Understand different ways of representing a function, that is, by using formulas, graphs, tables, and words.

Functions, Graphs, Solving, and Applications

1. Solve linear equations symbolically.
2. Understand and apply some basic properties of graphs of functions, such as intercepts, and intersections on the TI 83/84 calculator.
3. Solve equations using the TI 83/84 calculator.
4. Set up formulas for functions related to economic, geometric, and physical models. In particular, the student should be able to convert a simple verbal description of a function into a formula or graph of the function.
5. Understand some basic properties of graphs of functions, such as intercepts, increasing, decreasing, concave up and concave down, and what these properties mean in practical terms.
6. Solve optimization problems using the graphing calculator.

Linear Functions

1. Determine if a table of ordered pairs for a function can be modeled by a function.
2. Graph linear functions defined by a table of values.
3. Find the slope of a linear function defined by a table of values.
4. Find the slope of a linear function given its graph where two points are identifiable.
5. Find the slope of a linear function given an expression that defines the function.
6. Find the slope as rise/run for a given problem.
7. Find a formula for a linear function given a table of values for the function.
8. Find the *y*-intercept of a linear function given a table of values, a graph, or a defining expression.
9. Understand least squares as a way to find the line of "best fit" for a given set of ordered pairs.
10. Draw a scatter plot on a TI 83/84.
11. Find the regression equation for a set of data using the TI 83/84.
12. Use a regression equation to answer questions about related data.

Quadratic Functions

1. Be familiar with the general form of a quadratic function.
2. Understand the basic relationships between the equation and graph of a quadratic function.
3. Be familiar with determining *x* and *y* intercepts of a quadratic function both algebraically and with the aid of the calculator.
4. Be able to determine the roots of a quadratic function using the quadratic formula.

5. Be familiar with the relationship between the discriminant of a quadratic function and the types of roots of the quadratic function.
6. Be familiar with the vertex-form of a quadratic function.
7. Be familiar with quadratic data having constant second differences.
8. Be able to do quadratic regression on almost quadratic data.
9. Understand sum-of-squares error and the quadratic function of best fit.

Power and Polynomial Functions

1. Understand characteristics of the graphs of power functions.
2. Interpret and apply the concept of direct variation as an example of a power function.
3. Interpret and apply the concept of inverse variation.
4. Determine if a function is a polynomial function.
5. State the degree of a polynomial function.
6. Describe the end behavior of polynomial functions.
7. Determine the intercepts of a polynomial function.
8. Determine the domain and range of a polynomial function.
9. Find the local maximum and local minimum points of a polynomial function.
10. Find and use a cubic regression equation for a set of data.

Exponential Functions

1. Understand the concept of exponential growth.
2. Compute the growth factor of an exponential function and determine the growth rate.
3. Understand the relationship between the growth factor and exponential growth or decline.
4. Understand and apply the compound interest formulas for periodic and continuous compounding.
5. Determine whether a set of data can be modeled by an exponential function and construct a model.
6. Use an exponential function model.
7. Determine which of two models is a better fit.
8. Use exponential regression to determine an exponential function model.

Logarithmic Functions

1. Understand the concept of a logarithmic function.
2. Understand the concept of an inverse function.
3. Write exponential statements in logarithmic form.

4. Write logarithmic statements in exponential form.
5. Evaluate logarithms using a calculator.
6. Understand properties of logarithms.
7. Solve exponential equations.
8. Find and use a logarithmic regression equation for a set of data.

Piecewise-defined Functions

1. Understand the concept of a piecewise-defined function.
2. Work with piecewise-defined functions given by multiple formulas.
3. Be able to graph piecewise-defined functions with the calculator.
4. Be able to set up formulas for piecewise-defined functions.
5. Be able to model data with piecewise-defined functions.

Technical Mathematics Pathway

A few systems and college have developed technical mathematics courses. In Indiana, Ivy Tech Community College (n.d.) developed a technical mathematics pathway.[33] The pathway does have a prerequisite course, but the college-level course is not designed for transfer students. Learning outcomes for the courses are:

1. **Geometry: Upon completion, students will be able to complete the following:**
    a. Calculate and use perimeter, area, volume, and surface area to find same or unknown measures of parts.
    b. Graph linear equations in two dimensions; calculate and apply slope of a line.
    c. Identify the parts of a circle, including diameter, radius, circumference, sectors, and segments and to calculate the area of circles. Calculate volumes of regular cylinders.
    d. Perform common constructions using a straightedge and compass.
    e. Find angles of a regular polygon.
    f. Find the measures of complementary, supplementary, vertical, interior, exterior, and corresponding angles and calculate the sum of the measures of the interior angles of polygons.
    g. Use proportion as applied to similar figures.
    h. Apply the Pythagorean Theorem.

33 https://wwwapps.ivytech.edu/cor3/f/3/3/qtYgyGa2dTujpFRyUAOuo96DCPSEponaazX1EY3stwFCJq0IQQrlv0gQahkyeh3Y

2. **Trigonometry: Upon completion, students will be able to complete the following:**
   a. Determine the sine, cosine, and tangent of angles in right triangles from direct measurement of lengths of sides.
   b. Solve for unknown sides and angles in right triangles.
   c. Utilize the Law of Sines and Law of Cosines to solve for unknown sides and angles in triangles.
   d. Determine angle (degree/minute/seconds, radians) measures using inverse trigonometric functions.
3. **Conversion: Upon completion, students will be able to complete the following:**
   a. Recognize and operate within and between different measurement systems, including dimensional analysis.
4. **Statistics: Upon completion, students will be able to complete the following:**
   a. Use measures of central tendency and normal distributions to estimate and validate statistical measures in the workplace.
   b. Use tables, charts, and graphs in applications common to the workplace.
5. **Algebra: Upon completion, students will be able to complete the following:**
   a. Recognize, set up, and solve application problems using direct and inverse proportionality.
   b. Solve a variety of real-life application problems using algebraic, geometric, trigonometric, and statistical formulas by solving equations in one variable.
   c. Solve formulas for any variable.
6. **Math-Related Job Skills: Upon completion, students will be able to complete the following:**
   a. Translate verbal and written descriptions into mathematical statements that solve real-world problems.
   b. Estimate and calculate values for fabricating, manufacturing, and business examples.
   c. Use a scientific calculator proficiently as related to coursework.

This pathway aligns with such technology programs as Advanced Automation and Robotics and Design Technology, as well as HVAC and Industrial Apprenticeship.

Business Math Pathway

Business math is designed to prepare students for business applications. Texas is one of several states that has adopted a business pathway and one of the few that has developed outcomes at a statewide level. The learning outcomes for Mathematics for Business and Social Science at Texas community colleges, which are included in the Texas Higher Education Coordinating Board's Academic Course Guide Manual[34] (2018), are brief and high level (Texas Higher Education Coordinating Board, 2018, pp. 170–171).

- Apply elementary functions, including linear, quadratic, polynomial, rational, logarithmic, and exponential functions, to solving real-world problems.
- Solve mathematics of finance problems, including the computation of interest, annuities, and amortization of loans.
- Apply basic matrix operations, including linear programming methods, to solve application problems.
- Demonstrate fundamental probability techniques and application of those techniques, including expected value, to solve problems.
- Apply matrix skills and probability analyses to model applications to solve real-world problems.

Logic Pathway

In addition to the state of Washington's five main math pathways, higher education institutions in the state allow a five-unit symbolic logic course to meet the quantitative reasoning requirement. The state's direct transfer agreement specifies that the course should focus on "sentence logic with proofs" and "predicate logic with quantifiers and proofs and/or Aristotelian logic with Venn Diagrams" (Intercollege Relations Commission, 2017, p. 19).

Although this pathway isn't widely used for transfer, Washington State University (n.d.) offers a six-unit philosophy course[35] that meets the requirement. In addition to studying topics in the history of philosophy and developing the skills to articulate philosophical positions, according to the syllabus, students study

- basic concepts of logic
- categorical logic
- immediate inference involving basic categorical sentences
- Venn diagrams to determine the validity of categorical syllogisms
- compound sentences containing truth-functional connectives
- truth tables and derivations

34 http://reports.thecb.state.tx.us/ibi_apps/WFServlet.ibfs?PG_REQTYPE=REDIRECT&PG_MRsaved=false&PG_Func=GETBINARY&PG_File=blnkvldc.pdf

35 https://apps.aoi.wsu.edu/coursemoreinfo/syllabus.aspx?courseid=2018-spri-ONLIN-PHIL-201-8638-LEC

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